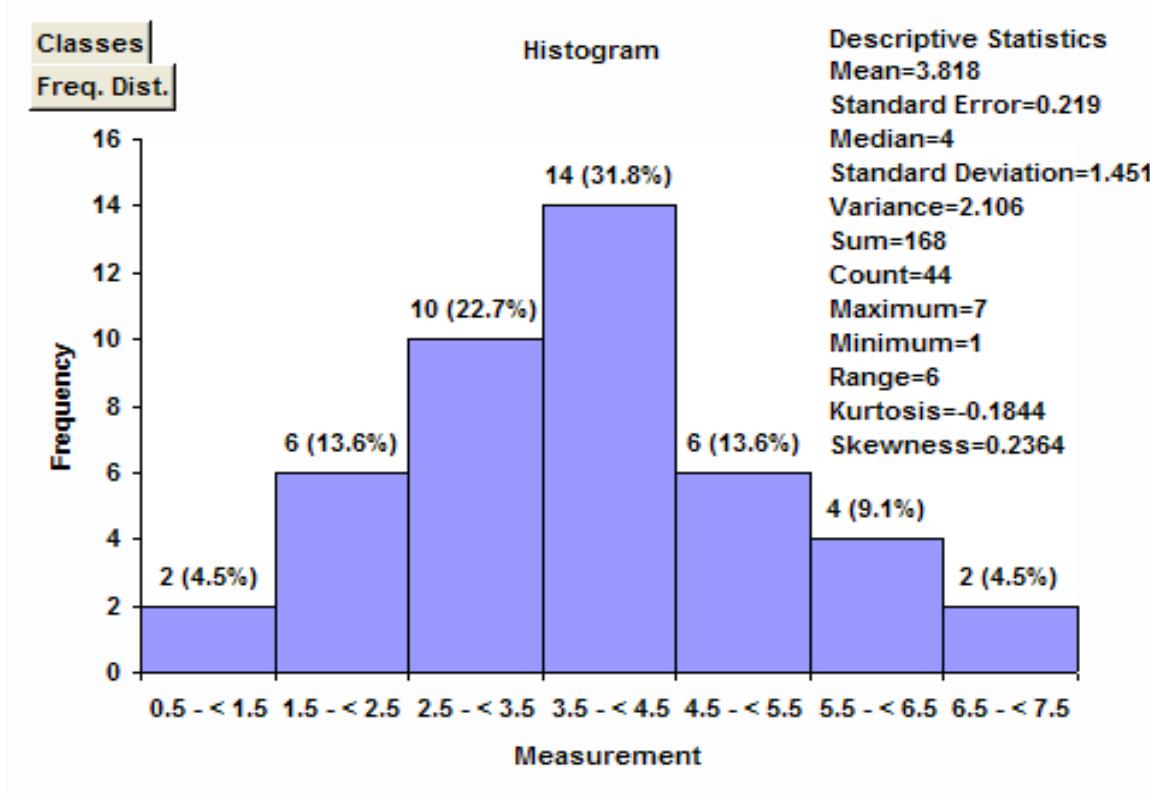


14.3 Measures of Dispersion



A measure of dispersion gives the spread of the data: clumped together or spread out.

The **range** is the difference between the largest and smallest value.

Ex 3 5 3 4 1 3 2 5 4

$$\text{Range} = 5 - 1 = 4$$

Better idea: how much does the data deviate from the mean?

Recall \bar{x} = mean

$x - \bar{x}$ = deviation from mean

Example:

Sample Data = 2, 4, 7, 5, 4, 8

Mean \bar{x} =

Data x	$x - \bar{x}$
2	
4	
7	
5	
4	
8	

Example:

Sample Data = 2, 4, 7, 5, 4, 8

Mean $\bar{x} = 30 / 6 = 5$

Data x	$x - \bar{x}$
2	-3
4	-1
7	2
5	0
4	-1
8	3

Notice: the sum of the deviations is 0

Idea: sum, then average, the squares!

Data x	$x - \bar{x}$	$(x - \bar{x})^2$
2	-3	
4	-1	
7	2	
5	0	
4	-1	
8	3	

Definition:

$$\text{Variance} = \frac{\sum (x-\bar{x})^2}{n-1}$$

Data x	$x - \bar{x}$	$(x - \bar{x})^2$
2	-3	9
4	-1	1
7	2	4
5	0	0
4	-1	1
8	3	9

$$\text{Sum } (\Sigma) \quad 0 \quad 24$$

Definition:

$$\text{Variance} = \frac{\sum (x-\bar{x})^2}{n-1}$$

Data x	$x - \bar{x}$	$(x - \bar{x})^2$
2	-3	9
4	-1	1
7	2	4
5	0	0
4	-1	1
8	3	9

$$\begin{aligned}\text{variance} \\ = 24/(6-1) \\ = 1.2\end{aligned}$$

$$\text{Sum } (\Sigma) \quad 0 \quad 24$$

Definition:

$$\text{Std. Deviation} = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

Data x	$x - \bar{x}$	$(x - \bar{x})^2$	
2	-3	9	variance
4	-1	1	$= 24/5$
7	2	4	$= 1.2$
5	0	0	
4	-1	1	std. dev.
8	3	9	$= (1.2)^{1/2}$
Sum (Σ)	0	24	$= 1.09544$

Example:

Data: 8 6 7 5 4

N =

mean \bar{x} =

Data x	$x - \bar{x}$	$(x - \bar{x})^2$
8		
6		
7		
5		
4		

Sum (Σ)

Example:

Data: 8 6 7 5 4

N = 5

mean $\bar{x} = 30/5 = 6$

Data x	$x - \bar{x}$	$(x - \bar{x})^2$	
8	2	4	variance
6	0	0	$= 10/(5-1)$
7	1	1	$= 2.5$
5	-1	1	
4	-2	4	std. dev.
Sum (Σ)	0	10	$= (2.5)^{1/2}$
			$= 1.5811$

Given a frequency table:

- N is the sum of the frequencies
- Find $(x - \bar{x})^2$ and multiply it by the frequencies
- If you wish, as a check, multiply $(x - \bar{x})$ by the frequencies. The sum should be 0.

The **median** of a set of data is the number in the middle of the list when the numbers are ordered.

If there is no single middle number, average the two middle numbers.

Data x	Freq f	xf	$(x - \bar{x})$	$(\bar{x} - x)^2$	$(\bar{x} - x)^2 f$
10	3				
11	1				
12	5				
13	6				
14	4				
15	1				

Sum (Σ)

Data x	Freq f	xf	$(x - \bar{x})$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
10	3	30	-2.5	6.25	18.75
11	1	11	-1.5	2.25	2.25
12	5	60	-0.5	0.25	1.25
13	6	78	0.5	0.25	1.50
14	4	56	1.5	2.25	9.00
15	1	15	2.5	6.25	6.25

Sum (Σ) 20 250 39.00

$$\text{Mean } \bar{x} = 250 / 20 = 12.5$$

$$\text{Var} = 39.00 / (20-1) = 2.0526$$

$$\text{Std. Dev} = (2.0626)^{1/2} = 1.4327$$